

Ex 1.7 page 38

(18) When $p = \$100$, $q = 5 \Rightarrow (p_1, q_1) = (100, 5)$
 and when $p = \$50$, $q = 25 \Rightarrow (p_2, q_2) = (50, 25)$
 we want q as a function of p .

$$q(p) = mp + b$$

$\Rightarrow p$ is independent, q is dependent
 $\therefore$ coordinates (p, q)

$$\text{Slope, } m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{25 - 5}{50 - 100} = \frac{20}{-50} = -\frac{2}{5}$$

$\therefore$ Equation is

$$q - q_1 = m(p - p_1)$$

$$\Rightarrow q - 5 = -\frac{2}{5}(p - 100) = -\frac{2}{5}p + 40$$

$$\Rightarrow q = -\frac{2}{5}p + 40 + 5$$

$$\Rightarrow q = -\frac{2}{5}p + 45$$

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OR $q - q_2 = m(p - p_2)$

$$\Rightarrow q - 25 = -\frac{2}{5}(p - 50) = -\frac{2}{5}p + 20$$

$$\Rightarrow q = -\frac{2}{5}p + 20 + 25$$

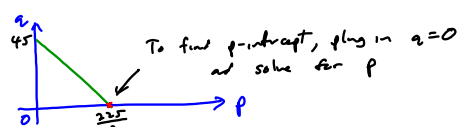
$$\Rightarrow q = -\frac{2}{5}p + 45$$

To find the inverse, we rewrite the above as p in terms of q .

$$\begin{aligned} \text{From, } q &= -\frac{2}{5}p + 45 & \Rightarrow p &= \frac{5}{-2}(-q + 45) \\ & \Rightarrow \frac{2}{5}p = -q + 45 & \Rightarrow p &= -\frac{5}{2}q + \frac{225}{2} \end{aligned}$$

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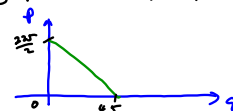
Down for $q = -\frac{2}{5}p + 45$



$$0 = -\frac{2}{5}p + 45 \Rightarrow \frac{2}{5}p = 45 \Rightarrow p = \left(\frac{5}{2}\right)(45) = \frac{225}{2}$$

$$p \in [50, 112.50]$$

$$q \in [0, 45]$$



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Chapter 2 Limits and Continuity

2.1 Marginal Cost and Marginal Revenue

We know that $C(q)$ is the total cost function of producing q no. of items.

If we increase our production by h no. of items, then we will be producing $q+h$ no. of items.

The total costs now become $C(q+h)$

So the increase in the total costs due to the extra production is

$$C(q+h) - C(q)$$

The average of this increase is

$$\frac{C(q+h) - C(q)}{h} \quad \Leftarrow \text{For every extra item, this is the extra cost incurred.}$$

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Example

If the cost of producing q no. of backpacks is $C(q) = 1000 + 50q + 0.2q^2$

and suppose that all this while we have been producing $q = 100$ backpacks per hour.

Our current costs are

$$C(100) = 1000 + 50(100) + 0.2(100)^2 = \$8,000$$

Let's assume we decide to increase our production by $h = 10$ backpacks per hour. Our new production costs become

$$C(q+h) = C(100+10) = C(110) \\ = 1000 + 50(110) + 0.2(110)^2 = \$8,920$$

The extra costs in producing an extra $h = 10$ backpacks

$$\text{is } C(q+h) - C(q) = \$8,920 - \$8,000 \\ = \$920$$

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On average, each extra backpack incurs an extra cost of

$$\frac{C(q+h) - C(q)}{h} = \frac{\$920}{10} = \$92 \text{ per backpack}$$

Suppose we change our minds and decide to produce an extra $h = 5$ backpacks per hour instead.

Our new costs become

$$C(q+h) = C(100+5) = C(105)$$

$$\Rightarrow C(105) = 1000 + 50(105) + 0.2(105)^2 = \$8,455$$

The extra costs incurred in producing an extra $h = 5$ backpacks is

$$C(q+h) - C(q) = \$8,455 - \$8,000 = \$455$$

And the average increase for each extra backpack

$$\text{is } \frac{C(q+h) - C(q)}{h} = \frac{\$455}{5} = \$91 \text{ per backpack}$$

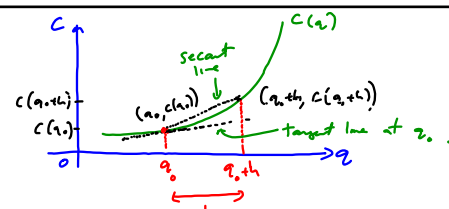
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Now, we ask ourselves what is the average extra cost if we decide to increase production by $h = 1$ backpack per hour

$$\frac{C(q+h) - C(q)}{h} = \frac{C(100+1) - C(100)}{1} = \frac{C(101) - C(100)}{1} \\ = \dots = \$90.20$$

We can see that the average increase in costs changes as the extra no. of items, h , changes.

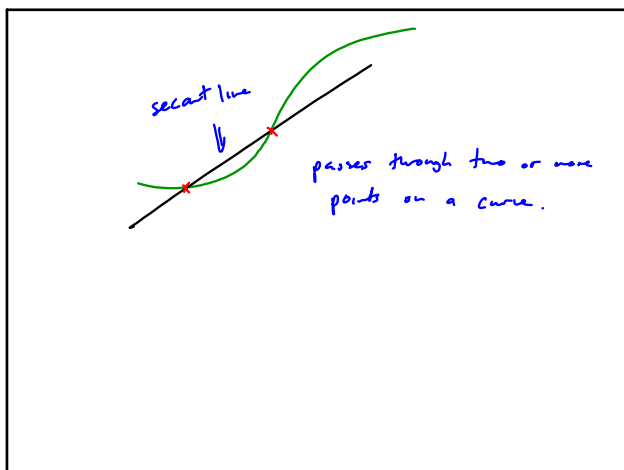


Average increase in producing an extra h no. of items

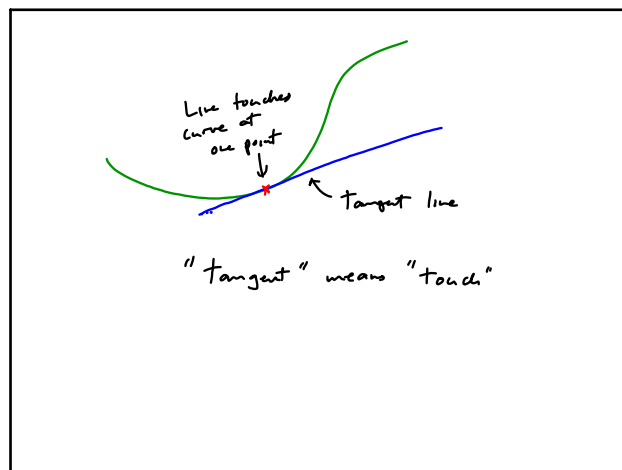
$$\text{is } \frac{C(q_0+h) - C(q_0)}{h} \leftarrow \text{Slope of the secant line passing } (q_0, C(q_0)) \text{ and } (q_0+h, C(q_0+h))$$

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As h approaches zero, i.e. $q_0 + h \rightarrow q_0$, the secant line approaches the tangent line at q_0 . And the slope of the secant line approaches the slope of the tangent line at q_0 . So, slope of tangent line at q_0 is

$$\lim_{h \rightarrow 0} \frac{C(q_0 + h) - C(q_0)}{h}$$

This is the instantaneous rate of change of $C(q)$ at q_0 .

The instantaneous rate of change of $C(q)$ at a general point q is called the Marginal Cost Function

$$MC(q) = \lim_{h \rightarrow 0} \frac{C(q+h) - C(q)}{h} \quad \Leftarrow \text{Derivative of } C(q)$$

Note Sometimes we also use the following notation:
 $MC(q) = \frac{dC}{dq} = C'(q)$ and other notations

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Marginal Revenue

Similarly, we obtain the Marginal Revenue Function from the q^{th} unit

$$MR(q) = \lim_{h \rightarrow 0} \frac{R(q+h) - R(q)}{h} \quad \Leftarrow \text{Derivative of } R(q)$$

Note

$$MR(q) = \frac{dR}{dq} = R'(q)$$

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In general, the derivative of $y=f(x)$ is

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

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Maximising Profit

The profit function is $P = R - C$

If at the q^{th} production and sales level,

① If $MC(q) < MR(q)$, then

if $q \uparrow$, then $P \uparrow$
 if $q \downarrow$, then $P \downarrow$

If $MC(q) < MR(q)$,
 we should increase
 production to increase
 profits

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② If $MC(q) = MR(q)$, then

profit is maximised at this production and sales level.

③ If $MC(q) > MR(q)$, then

if $q \uparrow$, then $P \downarrow$
 if $q \downarrow$, then $P \uparrow$

If $MC(q) > MR(q)$, then
 we should decrease production
 to increase profits.

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Exercise 2.1, pages 42 & 43

③ Given $R = 3q^2 - 10q$

Given $R(q) = 3q^2 - 10q$
 $\Rightarrow R(q+h) = 3(q+h)^2 - 10(q+h)$

$$\Rightarrow MR(q) = \lim_{h \rightarrow 0} \frac{R(q+h) - R(q)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(3(q+h)^2 - 10(q+h)) - (3q^2 - 10q)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(q^2 + 2qh + h^2) - 10q - 10h - 3q^2 + 10q}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3q^2 + 6qh + 3h^2 - 10h - 3q^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6qh + 3h^2 - 10h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(6q + 3h - 10)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} (6q + 3h - 10)$$

$$= 6q + 3(0) - 10 \quad \leftarrow \text{Direct substitution}$$

$$= 6q - 10 \quad \therefore \text{Given } R = 3q^2 - 10q$$

$$\Rightarrow MR(q) = 6q - 10$$

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